Fourier series

In mathematics, a Fourier series is a way to represent a wave-like function as the sum of simple sine or cosine waves. More formally, it decomposes any periodic function or periodic signal into the sum of a (possibly infinite) set of simple oscillating functions, namely sines and cosines (or, equivalently, complex exponentials as shown in Fig.1.

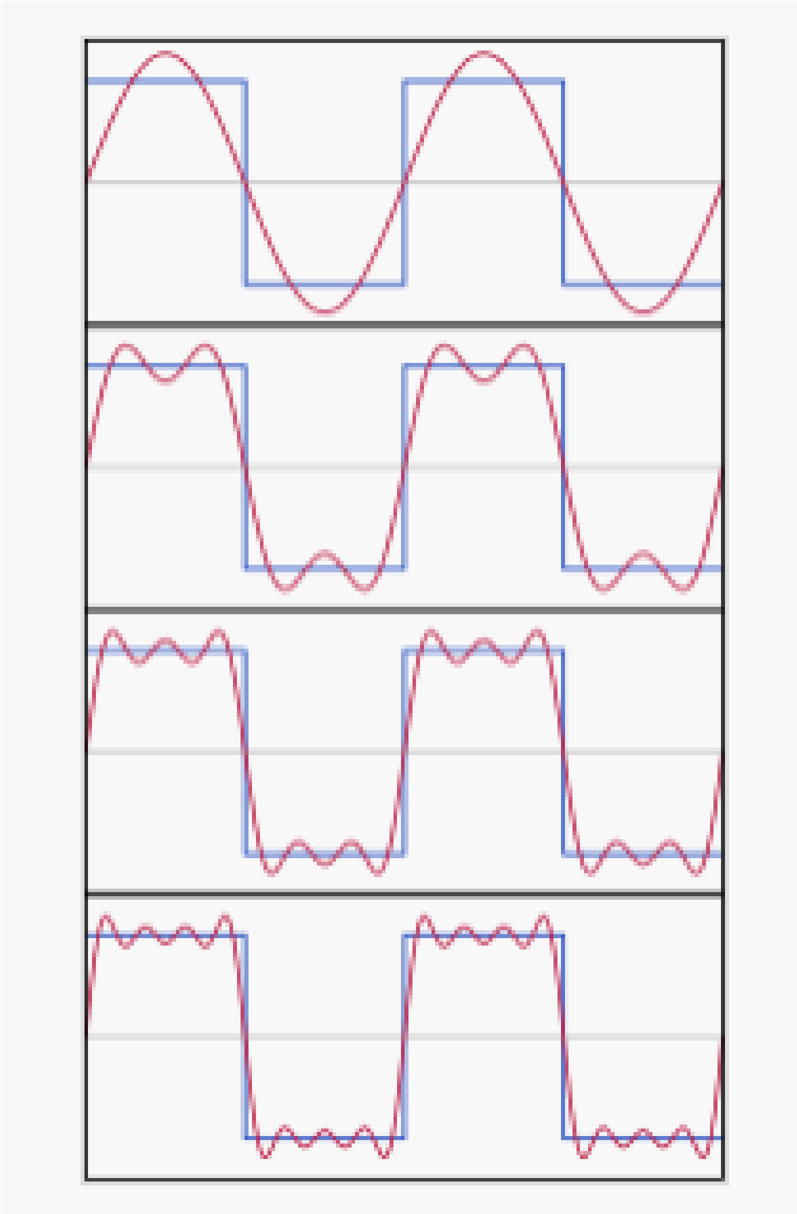

Figure (1): The first four partial sums of the Fourier series for a square wave.

The primary reason that we use Fourier series is that we can better analyze a signal in another domain rather in the original domain. Sometimes a signal reveals itself more in another domain.

$$F(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$$

Periodicity:

The first thing to notice about a Fourier series representation of a function is that the function must be periodic. A function f(x) is said to be Periodic when

$$F(x+T)=F(x)$$

for some value T as in Fig.2.

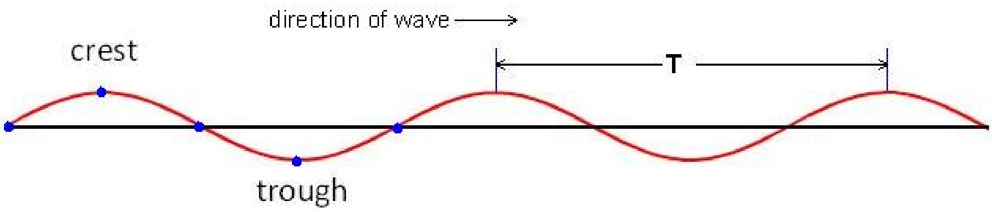

Figure (2): Periodic Function

Fourier series Forms:

1. Fourier series in real form:

Three coefficients used to describe Fourier series in real form

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx \qquad n = 1, 2, 3, \ldots$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx\, dx \qquad n = 1, 2, 3, \ldots$$

2. Fourier series in complex form

Fourier series can be expressed more simply using complex exponentials. Moreover, because of the unique properties of the exponential function, Fourier series are often easier to manipulate in complex form.

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-int} f(t)\, dt$$

$$c_0 = \tfrac{1}{2} a_0\,, \quad c_k = \tfrac{1}{2}(a_k - ib_k)\,, \quad c_{-k} = \tfrac{1}{2}(a_k + ib_k)\,.$$